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Determinism, Interpretations of Probability and the Rational Expectations

Introduction

The history of economic thought and the economic theory provide much evidence that scholars can't come to terms with the following fact: a course of market events is ruled by mechanism which can not be imitated – in the exact (quantitative) way or in terms of measurable probability – by any model.

With this in mind, we would like to emphasize that the acceptance and understanding of the nature of uncertainty play an important role in economic thought. Not because it enables a theoretician to make more accurate predictions of some market events, nor because it allows a practitioner, in the field of economics, to take more rational decisions. Above all, it is important because it enables them to differentiate between two spheres of economic analysis, namely within which the use of the probability calculus and sophisticated market models may really support their efforts to cope with an uncertain future, and the economic deliberation where uncertainty appears to be absolutely unavoidable.

In the first part of the paper we present the idea of determinism together with different interpretations of probability. All the time we try to refer the matter to the market. The second part deals examples of a widely used theory – the rational expectation hypothesis (REH). As a result we have come to the conclusion that REH works on a false assumption that all future events in socio-economic system can be viewed in terms of "class probability".

Economy, Science and Determinism

The man feels an instinctive concern for the unknown and unpredictable. Therefore, he stubbornly tries to give his reality features of fully stable, predictable and intentionally designed world. Initially, such visions were, by their nature, myths and religious beliefs. According to the ancient Greeks a course of events was just a predetermined gods' will. At that time philosophers insisted on the existence of "absolute truth" and didn't tolerate randomness. Later, in the medieval Europe, the course of history was unequivocally set by relative positions of celestial bodies. With the passing of time, visions of reality began to take on more and more scientific and formal shapes: in the field of the natural sciences, as well as in the social sciences, numerous attempts to construct the ideal model of the world have been undertaken.

During the "Great Scientific Revolution" (in the 17th and 18th centuries), scholars stopped perceiving natural phenomena as a result of an action of some (human or divine) consciousness and started explaining it in terms of aimless, impersonal forces. The crowning achievements of that era were Newton's laws of motion and Laplace's mechanistic vision of the world.

In the ideal Newton-Laplace's world there is no room for randomness, at least not in a way the result of tossing a coin is. Indeed, Laplace was arguing with deep conviction, that if the data defining locations and movements of all particles (molecules) of the world had been submitted to outstanding arithmetician, he would have – with the use of Newton's laws – perfectly predicted the future of the world[1]. Laplace's world, though it was broken free of gods' whims, was still deterministic. Determinism postulates that all events and processes in nature (or in the socio-economic system) proceed in a such way that the state of the world at any point in time – under some strict regularities – is a precondition of a world's state in another point in time[2].

While this point of view may be justified within physics[3], we can't find an excuses for "Laplace's attitude" represented by some economists. The deterministic neoclassical general equilibrium model played an important role in a debate, that was carried on in the twentieth century, between followers of the market and centrally planned economy. In its course Ludwig von Mises threw down the gauntlet to theorists of socialism: without private ownership there was no market for capital goods and without the market there

1 P.S. Laplace, *Philosophical essay on probabilities*, Springer Verlag, New York 1995.

2 S. Amsterdamski, Augustynek Z., Mejbaum W., *Prawo. Konieczność. Prawdopodobieństwo*, Książka i Wiedza, Warszawa 1964.

3 In laboratory, under an experimental conditions, often happens that an analyzed system is relatively simple and isolated, so for practical purposes it can be assumed that a physicist really knows everything about it.

were no prices. Therefore, an economic calculation – which is the basis for an efficient allocation of capital – was not possible[4]. Followers of the centrally planned economy demolished this argument, claiming that they were in possession of the economic model which allowed them to imitate the market without reference to actual market transactions. They believed that, with use of it, they would be able to determine where, when, and in what quantities concrete economic goods would find the best application. As they thought, it would be possible to simulate present and future prices, after all sources of valuable information about reconciliation between a production volume and a scale of request.

In the thirties August Friedrich von Hayek came to Mises' aid. He pointed out a very important and previously omitted aspect of economic analysis: in order to construct a perfect model of the market you must have perfect knowledge of the market[5].

The scientist, constructing an ideal model, would have to get to know the set of all parameters determining, in a correct and complete way, the state of our world in a certain moment. Just as physicians will never be able to collect information about locations of all the particles of the matter, economists will never get to know a complete set of factors characterizing the socio-economic system. In particular, it won't be accessible to them parameters which adequately reflect the tastes, preferences, knowledge and actions of individuals. They also won't be able to collect many other details that decide, for example, about the individual character of each of the devices used in the economy (for instance, details about wear of factories and goods involved in production throughout the economy[6]).

Furthermore, the assumption of a deterministic model – saying that researchers can find a definitive set of rules governing the development of either natural or social phenomena – is naive. Scientists discover new facts every day and formulate new theories about the reality around. Even Newton's discovery, so valuable and useful, as it turned out was not a timeless and absolute truth. At the end of the nineteenth century scientists began to observe facts which could not be explained using the concept developed by the classical mechanics[7]. Only later, on the basis of Einstein's theory of relativity, some of

4 L. von Mises, *Human Action. A Treatise on Economics*, Fox & Wilkes, San Francisco 1996.

5 F.A. von Hayek, *Individualism and Economic Order*, The University of Chicago Press, Chicago 1996.

6 *Ibidem*.

7 W. Sady, *Spór o racjonalność naukową. Od Poincarégo do Laudana*, book is available in electronic format at: http://www.scribd.com (originally published by FUNNA, Wroclaw 2000).

the observed anomalies were explained. The history of economic thought is full of similar examples: from time to time economists revise their views on the economy.

What's more, we shouldn't jump to conclusions that the processes of the nature or market processes take place in a deterministic way, that are "fixed in advance". Many scholars – including prominent twentieth-century physicists, Max Born and Niels Bohr – put forward a thesis about "indeterminism" prevailing in the nature. From what they claimed, it appeared that the starting point of explanation was randomness rather than causation. Therefore, an occurrence of specific events or phenomena needn't be the subject to any rule, but whole groups of events may reveal some statistical regularity, which can be described with the use of the theory of probability[8].

In this way, the sophisticated probability calculus and stochastic processes have appeared widely in the twentieth century physics, and the economics – as it was already happened before – followed physics' methods. However, economist didn't just "make a copy" of the physics – as it's claimed by some opponents of expressing social phenomena in the tool and methodological framework of the natural sciences – but they have, partly unconsciously, only appropriated some of physics' "external" manifestations. This has resulted in some theoretical and methodological problems in economics.

Interpretations of probability

Although an interest in probability has a long tradition (some elements of the theory were already applied to the analysis of gambling in the Middle Ages) a concept of probability still remains rather vague. In common parlance, "probability" indicates the possibility of a phenomenon or event. In the most common interpretation the term usually concerns one's expectation about result of some course of events which is unknown (at least to the person expressing the expectation). The basic feature of this "trivial" interpretation of probability is its relationship to everyday human life and actions. However, this intuitive interpretation is not fully consistent with the concept of probability related to physics, nor its completely axiomatic version predominated academic mathematics. These abstract categories – especially those of physics – have come to the fore in the social sciences.

8 S. Amsterdamski and others, *op. cit.*, W. Tatarkiewicz, *Historia filozofii*, vol. 3, Wydawnictwo Naukowe PWN, Warszawa 2009.

In the nineteenth and twentieth century probability calculus was one of the fastest developing areas of mathematics. The problem is that the improvement of probability theory, which still takes place, does not constitute a good argument for more and wider use of it, wherever it is actually being tried to apply. From a century-long philosophers dispute, which is not even close to the end, has come out a following conclusion: what is quite clear on the basis of mathematics axiomatic system, it becomes something very strange and inconsistent when we try to refer it to the real world. Although the range of logical and philosophical issues related to the interpretation of probability is very wide, physicists in the nineteenth century, ignoring some of these difficulties, successfully introduced the probability calculus to their deliberations in the natural sciences. The concept of probability related to the physics plays especially important role for social scientists because they adopted it unmodified, as they did earlier with the system of classical mechanics.

The area of applicability of the probability theory, considered in terms of physics, is limited to events and phenomena that can be expressed in statistical terms. This means that it is not possible to estimate a probability of occurrence of any event, but only those which are frequently repeated. This important limitation is a direct consequence of the fact that the probability calculus, commonly used by the representatives of science, is an expansion of so-called class probability (or probability of frequency). Essential feature of that approach is that it concerns only "mass" phenomena, not individual events.

The class probability is the ratio of numerical strength of one class (group) of events or phenomena to another class (group). For example, if the A group includes all the results of flipping of a coin obtained in very long series of tossing-up and the B group includes only those of flips which ended up falling eagle, then the B/A ratio numerically determines a chance of tossing an eagle. In other words, the class probability expresses relative frequency of events belonging to the first class (let's say – to the group of all results of tossing-up) and events belonging to the second class (i.e. to the group of eagles). So if the coin is symmetrical, the relative incidence of eagles among of all the results would be 1:2 and the risk of an eagle – measured by the ratio of classes' numerical strength – is ½. In the same way it can be determined a risk of many other events such as appearing "one" in throwing a dice. This method also allows to name a probability of choosing "a winning six" in a lottery[9] or to estimate a probability of occurrence of some

[9] Obviously, in this case we don't need to refer to an experiment since the class of events are pre-defined by game's rules. Probability of a win on the Polish lottery is 1:13.983.816.

phenomena, for example, during a process of chemical reaction in a laboratory experiment. The basic condition, however, is that all of examined situations must be repetitive events in their nature. For this reason there is no mathematical (or rather "physical") method that allows a researcher, for example, to calculate a probability of making a discovery of an effective cure for cancer till the end of present year.

Risk vs. risk

Since the uncertainty of future events is a necessary element of socio-economic system – at least because construction of a perfect deterministic model is beyond the reach of any theory – every future event can be usually considered in terms of probability rather, than in terms of Laplace's certainty. Due to it's popularity, the class probability is usaully identified with general notion of probability, which – as Mises remarked[10] – often leads to confusions.

In theoretical considerations regarding the occurrence of future events and phenomena in the market system economists frequently use the concepts of "risk". Two definitions of risk play a crucial role in their deliberations.

The first one says that risk is a kind of probability, which – in contrast to the uncertainty – undergoes mathematical estimates, so it can be accurately determined. This interpretation of the term "risk" was for the first time proposed by Frank Knight[11]. According to it, risk is just "physical" class probability, for which physicists have developed – "scientific" and just enough logically consistent – method of measurement, that has been discussed in the previous section.

Economists use the term "risk" also in another meaning, giving it additionally pejorative connotation. They mean a potential loss or – more generally – a result which is inconsistent with entrepreneur's expectations. Such use of that term doesn't refer to the nature of a probability the event (loss) may occur with. It is hard to find in literature a clear explanation of the relationship between "risk" in both meaning. Actually, it is much more easier to find evidence of authors' making a gaffe on account of this definitional turmoil. Sometimes it causes a misunderstanding or even an abuse, leading to mix up the concept of class probability (Knightian risk) and the case probability (Knightian uncertainty). As a result, it creates the illusion of certainty. For example,

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10 L. von Mises, chapter 6, *op. cit*.

11 F.H. Knight, *Risk, Uncertainty and Profit*, Houghton Mifflin, Boston 1921.

the term "currency risk" is not really "risk". In view of the foregoing, we should rather name it as "uncertainty of currency risk", since it refers to a possible currency rate change that leads to a result which is inconsistent with entrepreneurs' expectations, but at the same time it is impossible to calculate the probability of its occurrence. Therefore, it is necessary to bear in mind that the risk taken in the second sense means only the possibility of incurring loss in the future (for example, as a result of changing of macroeconomic parameters) but – despite "risk" in the name – a probability of that loss usually is not a subject of calculations, at least calculations that can be logically justified.

The probabilistic determinism

The concept of class probability is an important economic forecasting and research tool. Identification of its characteristics and the nature allows us to recognize its rather limited applicability. It seems to be in a conflict with the universality of its use.

The main differences between a deterministic neoclassical model and a contemporary "probabilistic" model usually come down to the fact that in the first one relationship between explanatory and explained variables are fixed and determined functionally, while in the second changes of function parameters or formulas are acceptable (but still predetermined!) as well as random components. For example, in macroeconomic models variables such as stock of money, GDP growth, interest rates and price level are often expressed as a random walk with a constant drift[12]. Because, after physics, "real" and fully "scientific" economic models are required to provide accurate quantitative estimates, they actually – without taking into account the probability other than class – only apparently differ from their neoclassical counterparts, as far as uncertainty and probability are concerned.

Economists, trying to obtain the above-described properties of the model, must fully specify in advance how market participants alter their decisions and how market aggregates – which ultimately reflect these decisions – change over time. Thus, both the schedule of these changes as well as the probability distribution of random factors included in the model must be predetermined. Recently, R. Frydman and M. Goldberg have drawn attention to this curiosity: "As different as they may appear, contemporary

[12] R. Frydman, Goldberg M., *Macroeconomic Theory for a World of Imperfect Knowledge*, "Capitalism and Society", Vol. 3: Iss. 3, 2008, Article 1 – see p. 31 in footnote.

probabilistic representations are as restrictive as their deterministic counterparts, linear or nonlinear: they presume that, conditional on his representation in the initial period, an economist can fully prespecify how an individual's decision making and aggregate outcomes change over time"[13].

Scientific explanation, which is behind such a model, must assume out of necessity that in the real world there is no creativity, people never learn or an economist is like a Laplace's demon. Specific example for this kind thinking is provided by models basing on the hypothesis of rational expectations.

Some notes on Game Theory

Before we proceed to the analysis of HRO, we would like to share a few observations on game theory, as its achievements seems to be important in the context of acceptance and understanding of uncertainty in economists' deliberations[14]. Game theory, created in 1944 by John von Neumann and Oskar Morgenstern, is often perceived as a tool for modeling economic phenomena in conditions of uncertainty.

The development of mathematical game theory has led to an unprecedented crop of economic concepts describing a process of decision-making of market agents, who remain in interaction and thus operate under conditions of imperfect knowledge and uncertainty. Through the prism of game theory, the situation of a particular agent is determined not only by decisions that he makes, but also by a random factor (game with the nature) and moves of other market participants – the players[15].

The very concept that market actors influence the market environment and stay each other in mutual interaction certainly goes a step further than the deterministic neoclassical theory: after all, human action is by its characterized by uncertainty, so that approach is apparently "compatible" with Knight's concept of uncertainty.

However, right after this important step towards taking an uncertainty into account is made, come serious regression, almost to the starting position: the formalization of

[13] R. Frydman, Goldberg M., *Imperfect Knowledge Economics – Exchange Rates and Risk*, Princeton University Press, Princeton and Oxford 2007, p. 64.

[14] Because we are focusing our attention on HRO here and due to the need for a limited length this paper we can include only a few selective comments on the game theory in it. We are aware of the fact that the rich legacy of game theory certainly deserves much more extensive discussion and we are going to devote next publication to it.

[15] Cf. S. Forlicz, *Niedoskonała wiedza podmiotów rynkowych*, Wydawnictwo Naukowe PWN, Warszawa 2001, p. 38–44.

the idea, in fact often forces the economists to impose restrictive conditions on the model. Consequently uncertainty and imperfect knowledge is in practice often replaced by an algorithm accurately identifying the movements of individual players and imposes very strict rules of interaction between them. As a result of such games deprecate the role of randomness and preclude an existence of uncertainty associated for example with the creativity of players (as a set of possible moves is predetermined and known to all).

One may say that above-mentioned complaint is now obsolete and at best, it can refer to the early "sequential" game theory models, which assumed full rationality of players who possess complete information about, among others, game rules, a set of strategies and payoff functions. The dynamic development of game theory actually resulted in the possibility of avoiding some of such limitations. For instance, there was introduced player ("nature"), that made his moves in a random manner (but this "randomness" was accurately described by a probability distribution thus it introduced into the model an additional element of risk, not uncertainty). John Harsanyi conducted fruitful research on games with incomplete information: in this kind of games players do not have full knowledge of the structure of the game (for example, of information hold by rivals, rivals' possible strategies, etc[16]).

We must admit that, in relation to the idea of such games, our argument loses some of its impact. However, we would like to note that the practical requirement of imposing a formal shape to these ideas results in a rigorous mathematical models that are capable of recognizing rather risk, not Knightian uncertainty.

For example, the Nash equilibrium in a game with incomplete information is achieved when each player chooses an optimal response to expected (with some probability) strategies of their rivals. In this type of games an equilibrium is usually determined by combinations of mixed strategies: each player's strategy consists of several possible choices of random nature and a fixed probability is assigned to every of choices (and a sum of these probabilities altogether is one). Thus, in the final instance, the matter is foundering from knowledge and completeness of distribution of certain random variable – during "randomization" uncertainty is usually replaced by risk.

16 Cf. A. Blajer-Gołębiewska, Zielenkiewicz M., *Teoria gier jako narzędzie ekonomii XX i XXI wieku*, [in:] *Teoretyczne aspekty gospodarowania*, ed. D. Kopycińska, Wydawnictwo Katedry Mikroekonomii Uniwersytetu Szczecińskiego, Szczecin 2005.

The rational expectations hypothesis

Among economists there is a general agreement that market participants, working up their plans and taking decisions, bear in mind their knowledge and expectations about future. Obviously, this is quite sensible ("rational") postulate and one does not deny it. However, what should be emphasized, it is not quiet correct description of the rational expectations hypothesis (as it is sometimes misleadingly suggested[17]). In fact, the rational expectations hypothesis (REH) is only a kind of assumption concerning the way of making expectations by individuals on market and it is aimed at enabling the economist to make a strict and quantitative macroeconomic forecast. In particular, it allows him to formally "gather" actors' expectations together in the form of an aggregate outcome, so that's why REH can be considered as a formal rule, as a matter of fact the most commonly used rule, of economic forecasting.

In 1961, John Muth, regarded as the father of the rational expectations theory, put forward the following hypothesis: "*expectations, since they are informed predictions of future events, are essentially the same as the predictions of the relevant economic theory*"[18]. In its cosmetically different version – by Robert Lucas and the new classics – it says that market participants' subjective expectations of economic variables correspond to true and objective expectations of these variables[19]. In other words, individuals behave as if they knew the "true" model of the market, so in their forecasts may occur only random errors that are independent of the information set available at the time when these expectations were formed. Therefore, on average, forecasts must be accurate because errors will cancel each other out, i.e., their expected value is zero. Furthermore, those expectations can not be systematically wrong – it is the important implication of REH. If did, market agents would learn from history and change the way they formed expectations, thereby eliminating systematic (biased) errors[20].

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17 Even in some academic textbooks. See, for instance, Hall R. E., Taylor J. B., *Macroeconomics* (fifth edition), W.W. Norton & Company, New York – London 1997.

18 J.F. Muth, *Rational Expectations and the Theory of Price Movements*, "Econometrica", Vol. 29, No. 3, July 1961, p. 316.

19 Cf. B. Snowdon, Vane H.R., *Modern Macroeconomics: its Origins, Development and Current State*, Edward Elgar, Cheltenham 2005, chapter 5.

20 *Ibidem*.

Behind postulated by REH probabilistic judgments that economists "imposed" on market agents (both in the field of predicting the future, as well as learning from the past), primarily lays the assumption of possibility of making such judgments. It implies that future course of action is predetermined, if not by any traditionally deterministic formula, then by probabilistic one (of course, in the meaning of class probability).

We would like to remind that an application of the class probability, in the form of probabilistic opinion about market or decision-making situations, to be logically justified requires a quantitative statement about stable relationship between groups (classes) of specific events occurring on the market. Otherwise, assuming, for instance, that the most of events are unique, there wouldn't be objective criterion by which the individual events could be attributed to one or another group (class). In this case, it wouldn't be possible to determine the relative frequency of the events of one group among events of the second group (simply because there would be no reference groups or they would contain only one/a few elements).

Thus, only under obviously false assumption of invariance (repeatability, lack of creativity, etc.) of economic conditions and events in the market, it would be possible to imagine a situation in which initial diversification of knowledge and information – a basis for the formulation of any forecast – in the end would be replaced by the "true" knowledge of the "real" economy model. Meanwhile, a situation on the market is mostly set by qualitatively new events, such as original designs created as a result of the innovative activities of entrepreneurs or business decisions (perhaps, except for the most routine ones). Attempts to predict events based on historic data, as Knight noticed, "*deal with situations which are far too unique* (…) *for any sort of statistical tabulations to have value for guidance*"[21]. The uniqueness is a result of human actions and creativity and causes the variability on the market which can not be pre-specified in advance either by an economist or by a market agent. After all, creative solutions and actions do not carry out previously existing rules, otherwise they wouldn't be creative. Past data, no matter how rich and how precise, may not include any information about the future, as well as the results of rolling different dices one thousand times don't reveal any information about the chance of "six" in the 1,001th. In this case, we have to deal with the probability of individual events (so called "case probability") which can not be quantified like class probability. In the market conditions of Knightian uncertainty there

[21] F.H. Knight, *op. cit.*, p. 232–233.

is even no basis for the development of the subjective "rational" expectations that would be coincident with some "objective" expectations and which can be expressed using mathematics and probability theory.

Summary and conclusions

Knight's observation – that the market often does not meet the assumptions of the theory of probability – for a long time hasn't enjoyed popularity, rather has been ignored. In the new economic literature, authors more often present the arguments that in real life situations market agents as well as economists have to deal with an immeasurable uncertainty far more often then with the risk. In the paper we have confronted this observation with the hypothesis of rational expectations, reaching the conclusion that its main drawback lies exactly in the implicit assumption about nature of "market probability".

Abstrakt

W niniejszym artykule zaprezentowaliśmy ideę determinizmu oraz alternatywne interpretacje prawdopodobieństwa i odnieśliśmy je do rzeczywistości rynkowej. Uważamy, że brak zrozumienia i akceptacji dla „prawdziwej" niepewności jest jednym z ważnych powodów tego, iż wyjaśnienia podawane przez teoretyków często nie odpowiadają rzeczywistości rynkowej, a czasami zaciemniają jej prawdziwy obraz. Z tej perspektywy oceniliśmy przypadek szeroko stosowanej teorii – hipotezę racjonalnych oczekiwań (HRO). Doszliśmy do wniosku, iż HRO opiera się na błędnym założeniu, że w systemie społeczno-gospodarczym rozwój wszystkich przyszłych zdarzeń może być postrzegany w kategoriach prawdopodobieństwa klas (ang. class probability).